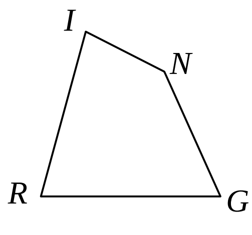
Geometry Notes TG - 9: Rigid Motions and Congruence

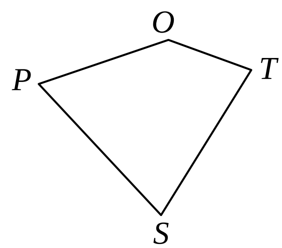
Congruence

Informal definition: Two polygons are *congruent* (≅) if they are the same shape and same size

Notation: In a statement of congruence of two polygons, the polygons are written so that corresponding (matching) vertices are in the same order.

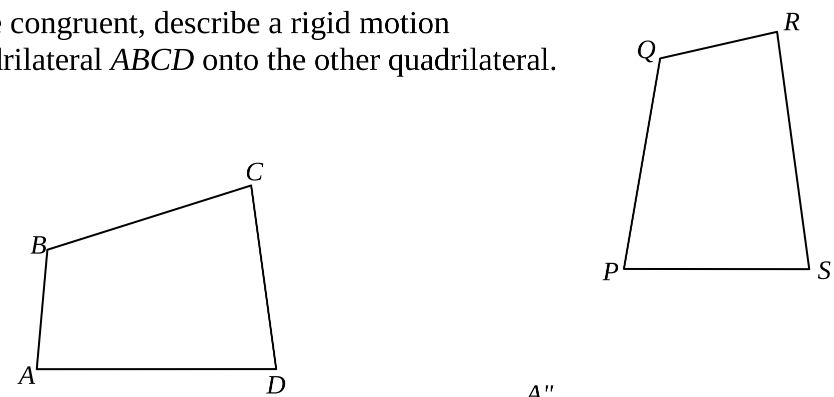


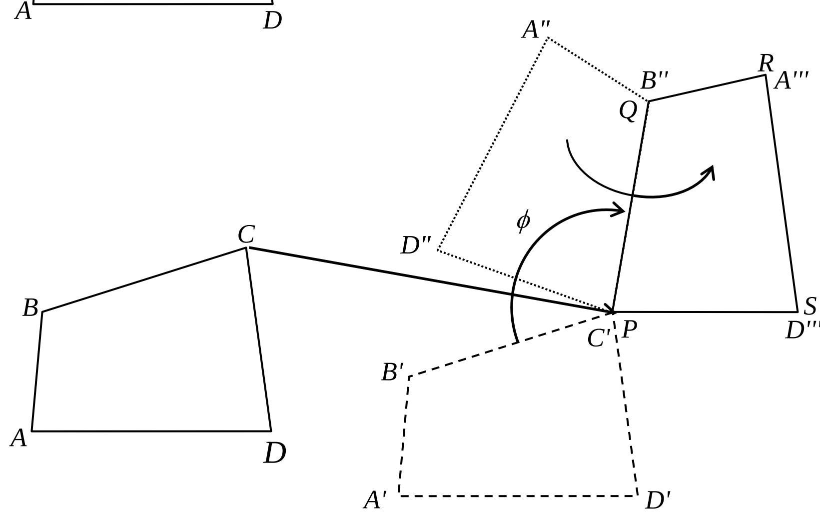
RING ≅ STOP



Definition: Two figures are *congruent* if one is the image of the other under a rigid motion.

Ex: Assuming they are congruent, describe a rigid motion that will take quadrilateral *ABCD* onto the other quadrilateral.





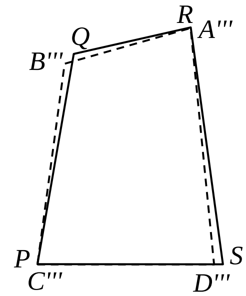
1) Translate along the vector $\overline{CP}$
2) Rotate CW until $\overline{B'C'}$ coincides with $\overline{QP}$
3) Reflect over $\overline{QP}$

Note: This was not the only possible way.

Note: For a polygon, congruence is only possible if

The vertices can be put into correspondence (matched) so that

1) All pairs of corresponding sides are congruent and
2) All pairs of corresponding angles are congruent.



Ex: If $\Delta DOG \cong \Delta CAT$,

a. Name three pairs of congruent angles.

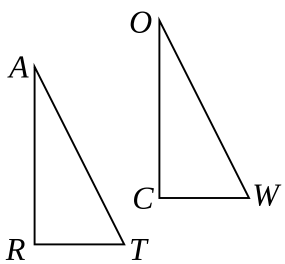
$\angle D \cong \angle C$, $\angle O \cong \angle A$, and $\angle G \cong \angle T$

b. Name three pairs of congruent sides.

$\overline{DO} \cong \overline{CA}$, $\overline{OG} \cong \overline{AT}$, and $\overline{DG} \cong \overline{CT}$

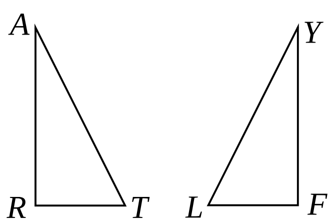
Ex: Describe a rigid motion that will take ΔRAT onto the other triangle.

a.



1) Translate along the vector $\overline{RC}$

b.

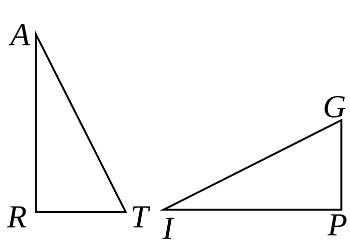


1) Translate along the vector $\overline{RF}$
2) Reflect over $\overline{FY}$ (image of $\overline{RA}$)

OR

1) Reflect over the vertical line that bisects $\overline{TL}$

c.

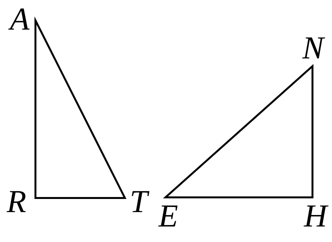


1) Rotate 90° CCW around point R
2) Translate along vector $\overline{RP}$

OR

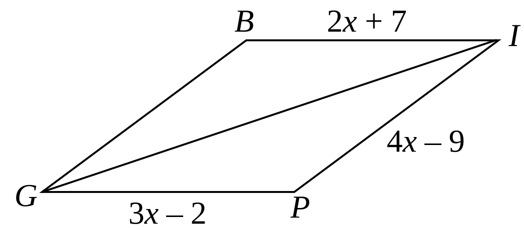
Reverse order (they're commutative)

d.



1) Translate along the vector $\overline{TN}$
2) Rotate CW until $\overline{TA}$ coincides with $\overline{NE}$
3) Reflect over the $\overline{NE}$

Ex: In the diagram at right $\Delta BIG \cong \Delta PIG$.
Find the perimeter of quadrilateral $BIPG$.



Since $\Delta BIG \cong \Delta PIG$, we know $\overline{BI} \cong \overline{PI}$.

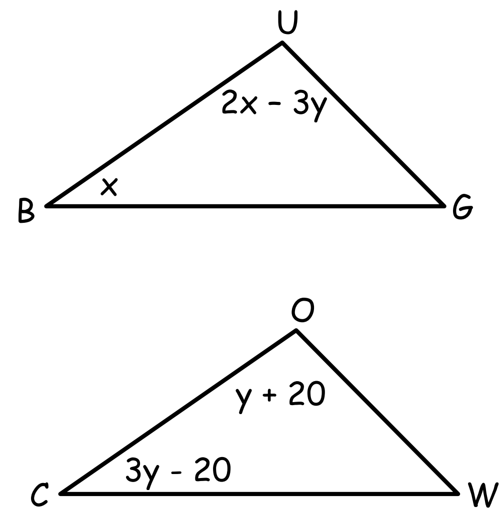
$$2x + 7 = 4x - 9$$
$$x = 8$$

$BI = PI = 2(8) + 7 = 23$

$BG = PG = 3(8) - 2 = 22$

Perimeter of BIPG $= 2(23) + 2(22) = 90$

Ex: If $\Delta BUG \cong \Delta COW$, $m\angle B = x$, $m\angle U = 2x - 3y$, $m\angle C = 3y - 20$ and $m\angle O = y + 20$, find the numerical measures of $\angle G$ and $\angle W$.



We know $\angle B \cong \angle C$ and $\angle U \cong \angle O$.

$x = 3y - 20$ and $2x - 3y = y + 20$

By substitution: $2(3y - 20) - 3y = y + 20$

$$6y - 40 - 3y = y + 20$$
$$2y = 60$$
$$y = 30$$
$$x = 3(30) - 20 = 70$$

$m\angle B = 70$, $m\angle U = 2(70) - 3(30) = 50$

$m\angle G = m\angle W = 180 - (70 + 50) = 60$