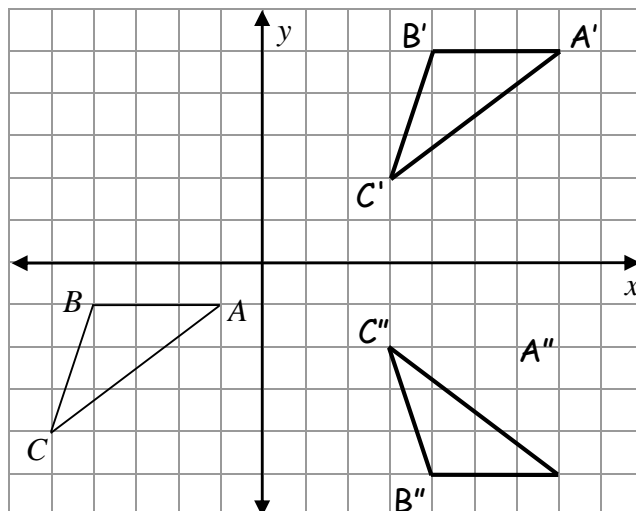


## Geometry Notes TG - 6: Compositions of Transformations

Ex:  $\triangle ABC$  has vertices  $A(-1, -1)$ ,  $B(-4, -1)$  and  $C(-5, -4)$ .

- a. Draw  $\triangle A'B'C'$ , the image of  $\triangle ABC$  after the transformation  $T_{8,6}$ .
- b. Draw  $\triangle A''B''C''$ , the image of  $\triangle A'B'C'$  after the transformation  $r_{x\text{-axis}}$ .

Definition: A *composition of transformations* is a combination of two (or more) transformations where the second transformation transforms the *image* of the first one.



Notation:  $r_{x\text{-axis}} \circ T_{8,6}(A) = A''$

Note: Compositions are done right to left. In above, do  $T_{8,6}$  FIRST, then  $r_{x\text{-axis}}$ .

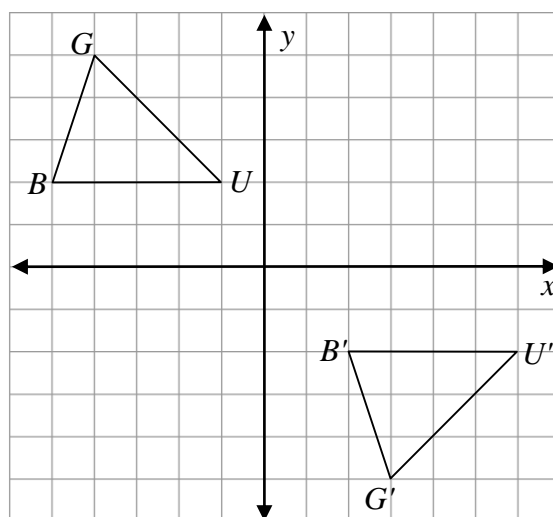
Ex:  $R_{90^\circ} \circ T_{2,-3}(2, 1) = R_{90^\circ}(4, -2) = (2, 4)$

Ex:  $T_{2,-3} \circ R_{90^\circ}(2, 1) = T_{2,-3}(-1, 2) = (1, -1)$

Note: In general, compositions are NOT commutative. (There are exceptions.)

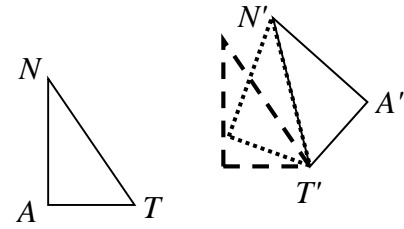
Ex: Write a composition of transformations that will take  $\triangle BUG$  onto its image  $\triangle B'U'G'$ . Note: There are several (actually infinite but let's not go there) possible answers.

- 1) Translate  $\triangle BUG$  right 7 units
  - 2) Reflect over the x-axis
- OR
- 1) Reflect  $\triangle BUG$  over the x-axis
  - 2) Translate right 7 units
- OR
- 1) Translate right 7, down 4
  - 2) Reflect over the line  $y = -2$



Ex: Describe a composition of transformations under which the image of  $\triangle ANT$  will be  $\triangle A'N'T'$ .

- 1) Translate along the vector  $\overline{TT'}$
- 2) Rotate CW about  $T'$  until  $\overline{TN}$  coincides with  $\overline{T'N'}$
- 3) Reflect over the line  $\overline{T'N'}$

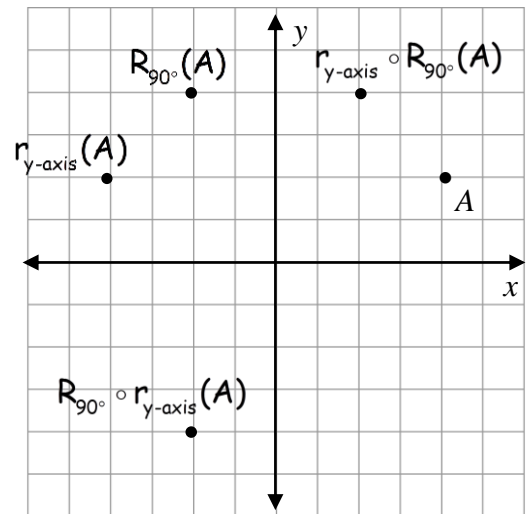


Ex: Is the composition  $r_{y\text{-axis}} \circ R_{90^\circ}$  commutative?

$$r_{y\text{-axis}} \circ R_{90^\circ}(x, y) = r_{y\text{-axis}}(-y, x) = (y, x)$$

$$R_{90^\circ} \circ r_{y\text{-axis}}(x, y) = R_{90^\circ}(-x, y) = (-y, -x)$$

No, this composition is not commutative.



Definition: A *glide reflection* is a composition of a line reflection and a translation in a direction parallel to the line of reflection.