

## Geometry Notes TG - 5: Dilations

### Dilations

A *dilation* is an enlargement or reduction of a figure.

If point  $P$  be the *center of dilation*, then

Enlargement: All points move **AWAY** from point  $P$   
by the same factor.

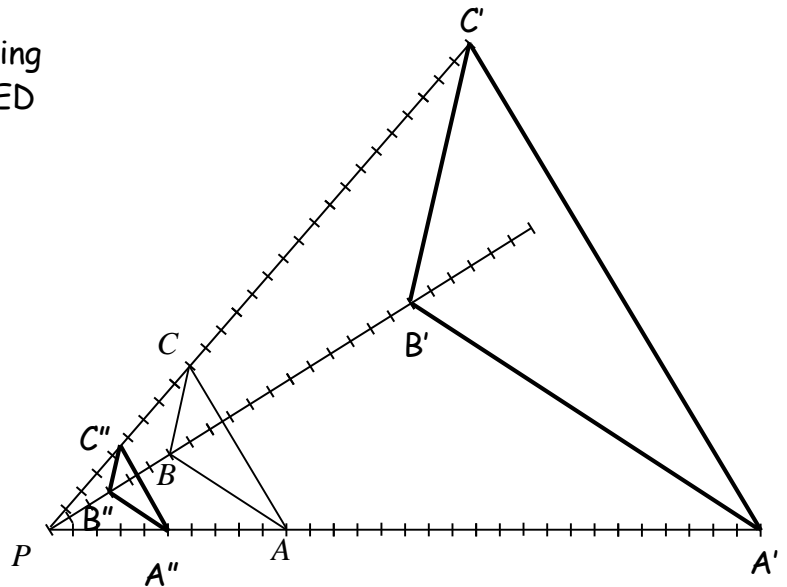
Remember: A factor is something  
that is **MULTIPLIED**

Reduction: All points move **TOWARD** point  $P$   
by the same factor.

The factor (or ratio) by which everything is  
enlarged or reduced is the *constant of dilation*,  $k$ .

Ex: In the figure at right,

- Find  $\Delta A'B'C'$ , the dilation of  $\Delta ABC$   
by a factor of 3 from the point  $P$ .  
(See graph)



$\Delta A'B'C'$  is similar to  $\Delta ABC$ . All distances (lengths) in  $\Delta A'B'C'$  are 3 times the corresponding distances in  $\Delta ABC$ .

Ex: If  $AC = 8$ , then  $A'C' = (3)(8) = 24$

- Find  $\Delta A''B''C''$ , the dilation of  $\Delta ABC$   
by a factor of  $1/2$  from the point  $P$ .  
(See graph)

Again,  $\Delta A''B''C''$  is similar to  $\Delta ABC$ . All distances (lengths) in  $\Delta A''B''C''$  are  $1/2$  the corresponding distances in  $\Delta ABC$ .

Ex: If  $AC = 8$ , then  $A''C'' = (1/2)(8) = 4$

## Dilations with Coordinates

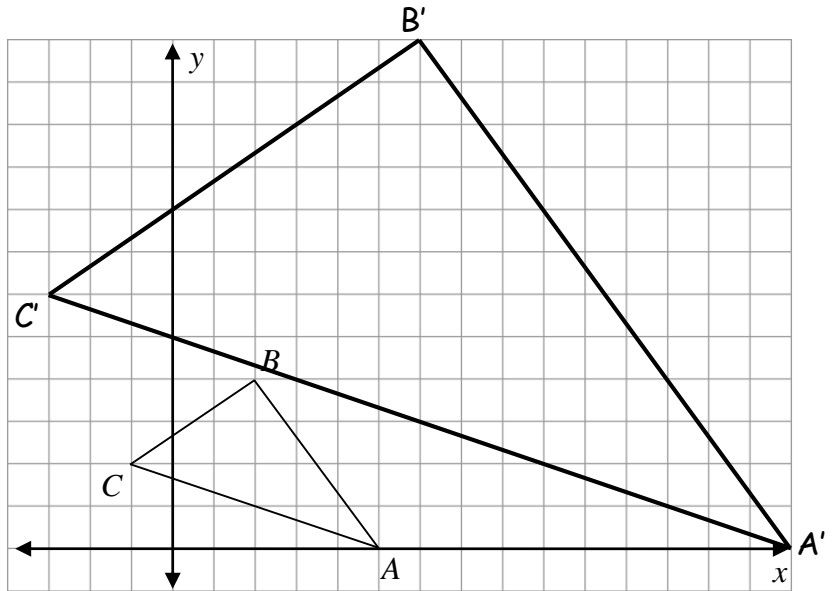
Ex:  $\triangle ABC$  has vertices at  $A(5, 0)$ ,  $B(2, 4)$  and  $C(-1, 2)$ . Dilate  $\triangle ABC$  by a factor of 3 from the origin.

$$A(5, 0) \rightarrow A'(15, 0)$$

$$B(2, 4) \rightarrow B'(6, 12)$$

$$C(-1, 2) \rightarrow C'(-3, 6)$$

In this dilation,  
 $P(x, y) \rightarrow P'(3x, 3y)$



$$\text{Notation: } D_3(x, y) = (3x, 3y)$$

In general,  $D_k(x, y) = (kx, ky)$  For dilations, think MULTIPLY.

Ex: 1)  $D_4(-3, 5) = (-12, 15)$

2)  $D_{3/4}(8, -12) = (6, -9)$

3) Find the value of  $k$  if  $D_k(6, -9) = (10, -15)$

$$6k = 10 \rightarrow k = 10/6 \text{ or } 5/3$$

Note that using the  $y$ -coordinates gives the same result.

4)  $D_{-2}(2, 3) = (-4, -6)$

A dilation by a negative constant is a combination of

1. A "normal" dilation and
2. A reflection in the origin.

