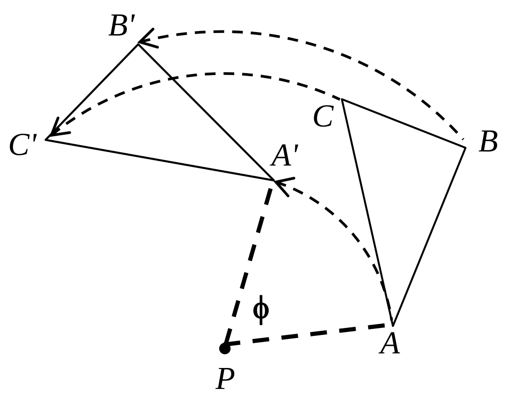
Geometry Notes TG - 2: Rotations

<u>Rotations</u>

Informal definition: All points rotate around a given point P by a given angle ϕ (does not have to be 180°).



Informal definition: All points rotate around a given point P by a given angle ϕ.

Notation: $R_{P,\phi}(A) = A'$ where P is the "center of rotation" and ϕ is the angle of rotation.

$m\angle APA' = \phi = m\angle BPB' = m\angle CPC' =$ etc.

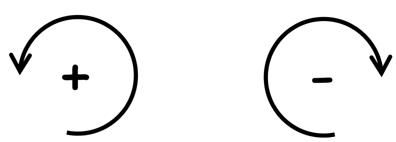
Properties of rotations:

1. The center of rotation P is invariant (unchanged)

2. For all other points, Q, the image Q' is the point such that

   $m\angle QPQ' = \phi$ $m\angle APA' = \phi = m\angle BPB' = m\angle CPC' =$ etc.
   and
   $PQ = PQ'$ $PA = PA'$, $PB = PB'$, $PC = PC'$, etc.

   Note: By definition, positive rotations are always counterclockwise

   Ex: For a counterclockwise rotation of 60°, write either 60° or 60° CCW

   For a clockwise rotation of 60°, write either -60° or 60° CCW (not -60° CW)

as a 180° rotation about the point.

3. Distances are preserved.

4. Angle measure is preserved.

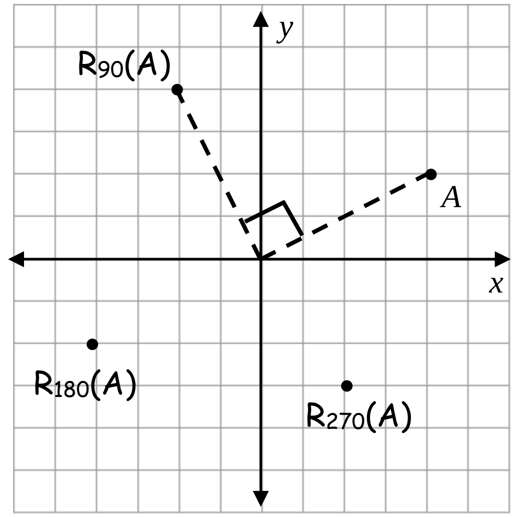
Special cases:

a. R_ϕ (no point specified) means rotate ϕ° around the origin

b. R_P (no angle specified) means rotate 180° around point P.

Origin Rotations with Coordinates

Ex: Let A have coordinates (4, 2).



a. $R_{90^\circ}(4, 2) = (-2, 4)$

$R_{90^\circ}(x, y) = (-y, x)$

b. $R_{180^\circ}(4, 2) = (-4, -2)$

$R_{180^\circ}(x, y) = (-x, -y) = R_O(x, y)$

c. $R_{270^\circ}(4, 2) = (2, -4)$

$R_{270^\circ}(x, y) = (y, -x)$

Note: R_{270} is the same as R_{-90}.

Note: It is probably NOT worth memorizing the formulas for R_{90} and R_{270}. Instead, to visualize each rotation, turn your whole paper by the appropriate amount.