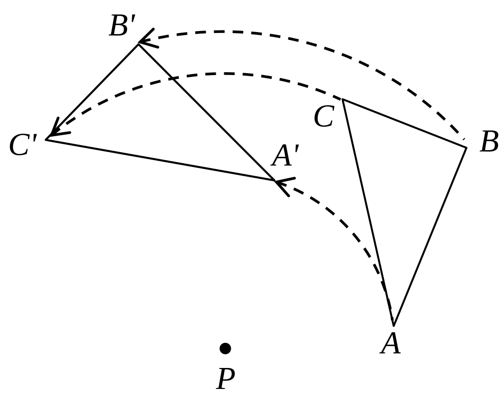
Geometry Notes TG - 2: Rotations

<u>Rotations</u>

Informal definition: All points rotate around a given point *P* by a given angle ϕ (does not have to be 180°).



Informal definition: All points rotate around a given point *P* by a given angle ϕ.

Notation:

Properties of rotations:

1. The center of rotation *P*

2. For all other points, *Q*, the image *Q′* is the point such that

Note: By definition, positive rotations are always

Ex: For a counterclockwise rotation of 60°, write either

For a clockwise rotation of 60°, write either

3. Distances are preserved.

4. Angle measure is preserved.

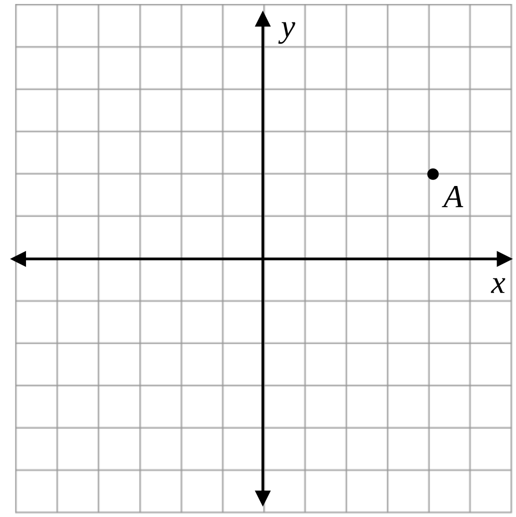
Special cases:

a. R_ϕ (no point specified) means rotate $\phi°$ around

b. R_P (no angle specified) means rotate around point *P*.

Origin Rotations with Coordinates

Ex: Let A have coordinates (4, 2).



a. $R_{90°}(4, 2) =$

$R_{90°}(x, y) =$

b. $R_{180°}(4, 2) =$

$R_{180°}(x, y) =$

c. $R_{270°}(4, 2) =$

$R_{270°}(x, y) =$

Geometry HW: Transformations – 2 Rotations

1. What is the image of (–3, 1) under a rotation of 90° about the origin?

2. a. What is the image of (4, –5) under a rotation of 180° about the origin?

   b. What is the difference between this and a rotation –180° (i.e. 180° CW) about the origin?

3. Using the rule $(x, y) \rightarrow (-y, x)$, find the image of $A(5, -2)$.

4. a. Find the coordinates P', the image of $P(x, y)$ after a reflection in the x-axis.

   b. Find the coordinates P'', the image of P' after a reflection in the y-axis.

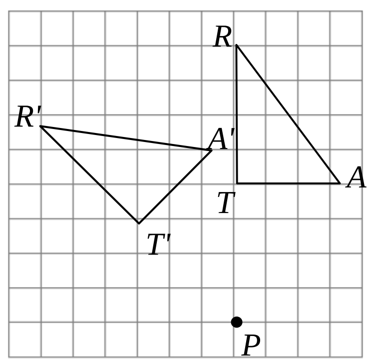
   c. A reflection in the x-axis followed by a reflection in the y-axis is the same as what single transformation?

5. a. **Graph** ΔRAT having coordinates $R(0, 2)$, $A(2, 5)$ and $T(5, 2)$.
   b. Graph $\Delta R'A'T'$, the image of ΔRAT after a 90° rotation about the origin.
   c. Graph $\Delta R''A''T''$, the image of ΔRAT after a reflection in the line $y = x$.

6. In the diagram at right, $\Delta R'A'T'$ is the image of ΔRAT after a rotation around point P.
   a. What is the angle and direction of rotation? (You do not need a protractor, just your brain.)

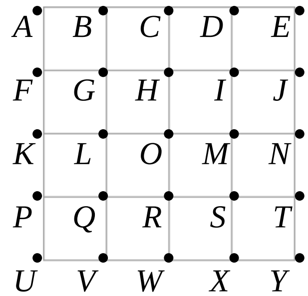
   b. What is the length of $\overline{R'T'}$? How do we know?

   c. What is the measure of $\angle R'T'A'$? How do we know?



7. Evaluate the following:
   a. $r_{y\text{-axis}}(3, -4) =$ b. $R_{180°}(4, 3) =$ c. $R_{90°}(0, 2) =$
   d. $R_O(3, -2) =$ e. $r_{y=x}(-5, -7) =$

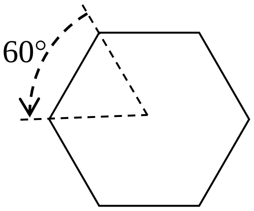
8. Refer to the diagram at right to answer the following:
   a. $R_{H,90°}(C) =$ b. $R_{R,90°}\left(\overline{TY}\right) =$ c. $R_L(P) =$
   d. $R_{O,90°}(\angle IJN) =$ e. $R_O\left(\overline{VW}\right) =$ f. $R_{O,270°}(D) =$
   g. $R_G(K) =$ h. $r_{\overline{KN}}(B) =$ i. $r_{\overline{PD}}(\overline{AB}) =$



Read: Rotational Symmetry and Point Symmetry

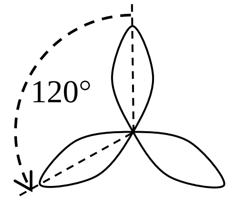
A figure has rotational symmetry if it is the image of itself after a rotation of $0° < \phi < 360°$.

Ex:



A regular hexagon has 60° rotational symmetry b/c it is its own image after a 60° rotation.
It also has rotational symmetry of all multiples of 60°: 120°, 180°, 240° and 300°.
Because it has 180° rotational symmetry, it is also said to have *point symmetry*.

Ex:



A “three leafed rose” has 120° rotational symmetry b/c it is its own image after a 120° rotation.
It also has rotational symmetry of all multiples of 120°: 240°.
Because it does *not* have 180° rotational symmetry, it does not have point symmetry.

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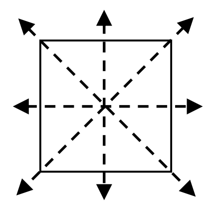
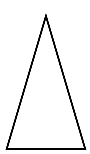
9. Which of the following letters has point symmetry but not line symmetry?
(1) **W** (2) **X** (3) **Y** (4) **Z**

Read: The Identity Symmetry

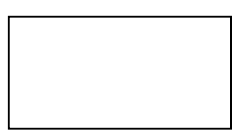
A rotation of 360° (or 0°) is called the “identity symmetry.” All figures have identity symmetry. When we count *total* symmetries, we include all lines of symmetry and all rotational symmetries and the identity symmetry. For example, a square has 8 total symmetries: Four lines of symmetry, shown, and four rotational symmetries (including the identity symmetry), 90°, 180°, 270° and 360°.



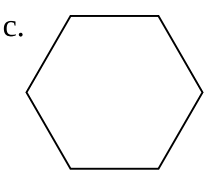
10. Tell how many total symmetries (including the identity symmetry) each figure has.

a. 

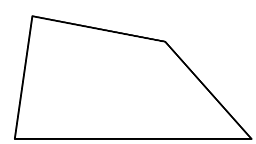
Isosceles triangle

b. 

Rectangle

c. 

Regular hexagon

d. 

Trapezium