Geometry Notes TG - 1: Line Reflections

Geometry Transformation

A *transformation* in geometry is a *mapping* of the points in the plane (think new seating chart for an infinitely large class), such that

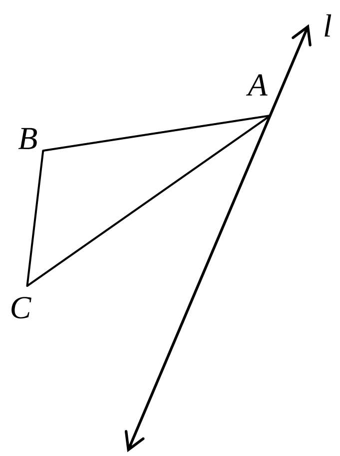
1. Each point P in the plane (called the pre-image), maps to a *unique* point P' (the image).

2. No two pre-image points have the same image.

3. Lines are preserved: If three points, P, Q, and R, are collinear, their images, P', Q', and R', will also be collinear.

(Line) Reflection

Ex: Sketch the reflection of ΔABC over the line l.



Informal Definition: In a line reflection over line ℓ, the image P' of each point P is found by

Notation:

Properties of line reflections:

1. Points *on* the line l

2. For points *not* on line l, l is of the segment from the point to its image.

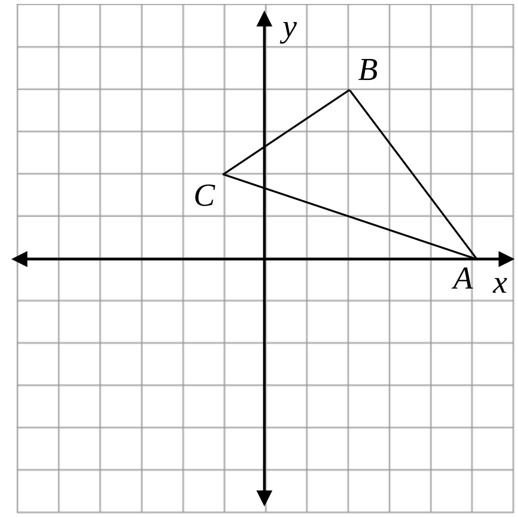
3. Distances are preserved.

4. Angle measure is preserved.

Line Reflections with Coordinates

Ex: ΔABC has vertices at $A(5, 0)$, $B(2, 4)$ and $C(-1, 2)$.

a) Draw $\Delta A'B'C'$, the image of ΔABC after a reflection in the x-axis.



$A(5, 0) \rightarrow A'$

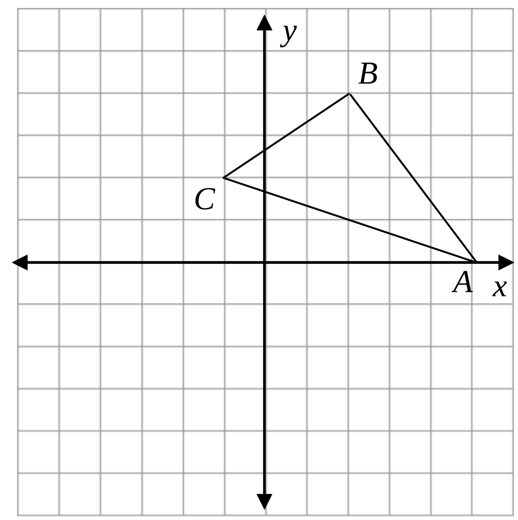
$B(2, 4) \rightarrow B'$

$C(-1, 2) \rightarrow C'$

In general, for a reflection in the x-axis:

$r_{x\text{-}axis}(x, y) =$

b) Draw $\Delta A'B'C'$, the image of ΔABC after a reflection in the y-axis.



$A(5, 0) \rightarrow A'$

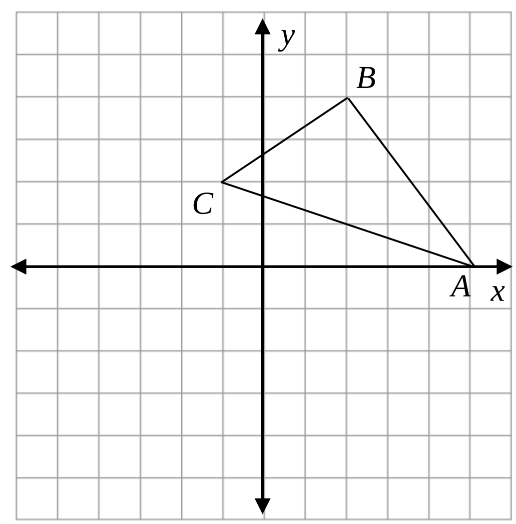
$B(2, 4) \rightarrow B'$

$C(-1, 2) \rightarrow C'$

In general, for a reflection in the y-axis:

$r_{y\text{-}axis}(x, y) =$

c) Draw $\Delta A'B'C'$, the image of ΔABC after a reflection in the line $y = x$.



$A(5, 0) \rightarrow A'$

$B(2, 4) \rightarrow B'$

$C(-1, 2) \rightarrow C'$

In general, for a reflection in the line $y = x$:

$r_{y=x}(x, y) =$

Geometry HW: Transformations – 1 ***Use Graph Paper!!

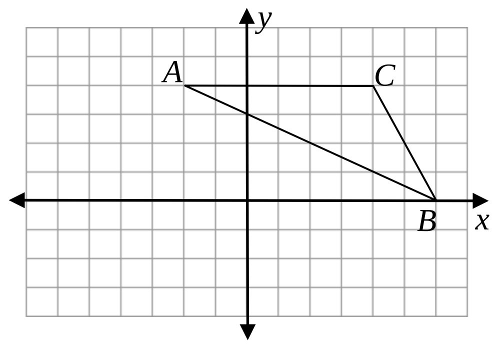
1. Find the coordinates of the image of the point (2, –7) under each of the following:
   a. $r_{y\text{-axis}}$. b. $r_{y=x}$. c. $r_{x\text{-axis}}$.

2. If the point (3, –1) is reflected in the *x*-axis and then that image is reflected in the *y*-axis, what are the coordinates of the final image?

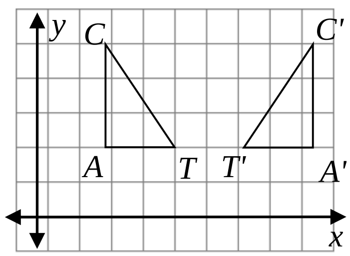
3. What are the coordinates of the image of the point (4, 1) after a reflection in the line $y = 3$?

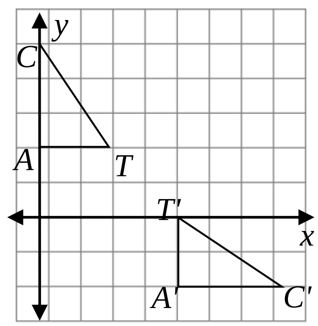
4. The image of the point *A*(–3, 1) after a reflection in line *k* is (7, 1). Find the equation of line *k*.

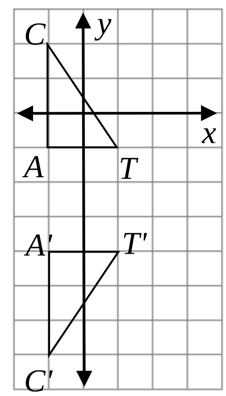
5. Triangle *ABC* is shown in the graph at right.
   a. Which point on the triangle will be invariant under a reflection in the *x*-axis?
   b. Give the coordinates of the points on the triangle that will be invariant (unchanged) under a reflection in the *y*-axis. (Invariant points are often also called *fixed points*.)
   c. Give the coordinates of the points on the triangle that will be invariant under a reflection in the line $y = x$.



6. For each diagram below, $\Delta C'A'T'$ is the image of ΔCAT after a line reflection. Write the equation of the line of reflection.

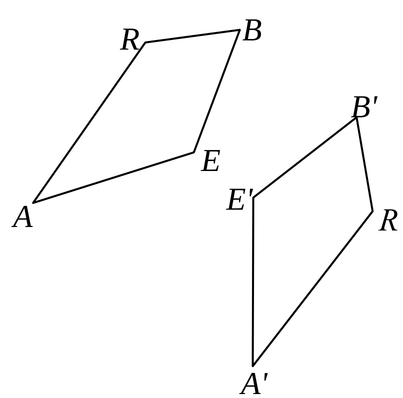
a.


b.


c.


7. Consider the diagram at right of *BEAR* and its image *B'E'A'R'*.
   a. Suppose we are told that *B'E'A'R'* is the image of *BEAR* after a line reflection. Describe briefly but precisely how we could find the line of reflection. HINT: Consider property 2 of a reflection.

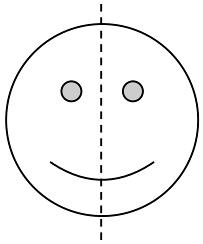
   b. Suppose we are *not sure* if *B'E'A'R'* is the image of *BEAR* after a line reflection. Describe briefly but precisely how we could find out if it is.



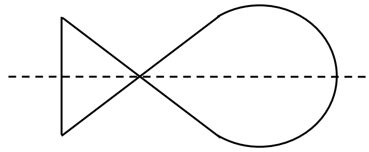
(This assignment is continued on the next page.)

Read: Line Symmetry

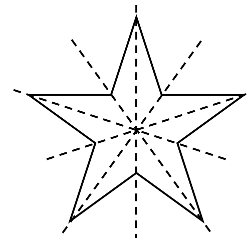
A figure has line symmetry if it is its own image after a line reflection. In middle school terms, the figure can be folded along a line and the two halves will match up exactly.

Ex: 

One vertical line of symmetry

Ex: 

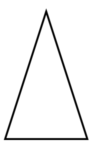
One horizontal line of symmetry

Ex: 

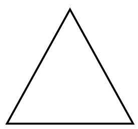
Five total lines of symmetry

8. Which letter has exactly one line of symmetry?
   (1) **H** (2) **I** (3) **J** (4) **K**

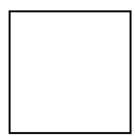
9. Tell how many lines of symmetry each figure has.

a. 

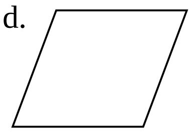
Isosceles Triangle

b. 

Equilateral Triangle

c. 

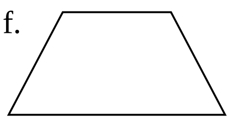
Square

d. 

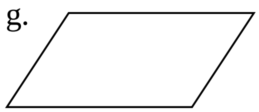
Rhombus

e. 

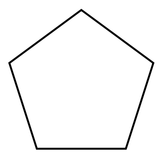
Rectangle

f. 

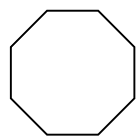
Isosceles Trapezoid

g. 

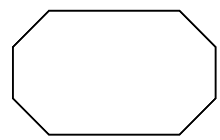
Parallelogram

h. 

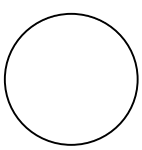
Regular Pentagon

i. 

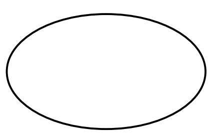
Regular Octagon

j. 

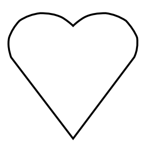
Octagon

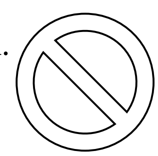
k. 

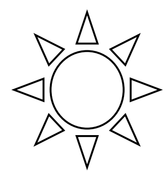
Circle

l. 

Ellipse

m. 

n. 

p. 

10. a. Graph $\overline{AB}$ with endpoints $A(1, 5)$ and $B(6, 3)$.
    b. Graph $\overline{A'B'}$, the image of $\overline{AB}$ under a reflection in the line $y = x$.
    c. Show using coordinate geometry that $\overline{AB} \cong \overline{A'B'}$.
    d. Show using coordinate geometry that the line $y = x$ is the perpendicular bisector of $\overline{AA'}$