Geometry Notes TG - 1: Line Reflections

Geometry Transformation   Discuss seating chart example.

A *transformation* in geometry is a *mapping* of the points in the plane (think new seating chart for an infinitely large class), such that

1. Each point *P* in the plane (called the pre-image), maps to a *unique* point *P'* (the image).

     No student moves to two different new seats.
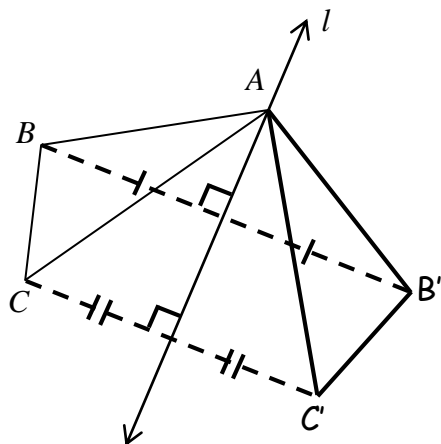
2. No two pre-image points have the same image.

     No two students move to the same new seat.

3. Lines are preserved: If three points, *P*, *Q*, and *R,* are collinear, their images, *P'*, *Q'*, and *R',* will also be collinear.

     Students in a line in the old chart are still in a line in the new chart.

Line Reflections

Ex: Sketch the reflection of Δ*ABC* over the line *l*.

For a line reflection, each point moves
1) Straight to the line (⊥) and then
2) Same distance beyond the line.

Notation: $r_l(B) = B'$

Pre-image   Image

Properties of line reflections:

1. Points *on* the line *l* don't move: $r_l(A) = A$ "Invariant" point.

2. For points *not* on line *l*, *l* is the perpendicular bisector of the segment from the point to its image.

     l is the perpendicular bisector of $\overline{BB'}$, $\overline{CC'}$, etc.

3. Distances are preserved.   Lengths of segments are unchanged.

4. Angle measure is preserved.   Angle measures are unchanged.

Line Reflections with Coordinates

Ex: $\triangle ABC$ has vertices at $A(5, 0)$, $B(2, 4)$ and $C(-1, 2)$.

a) Draw $\triangle A'B'C'$, the image of $\triangle ABC$ after a
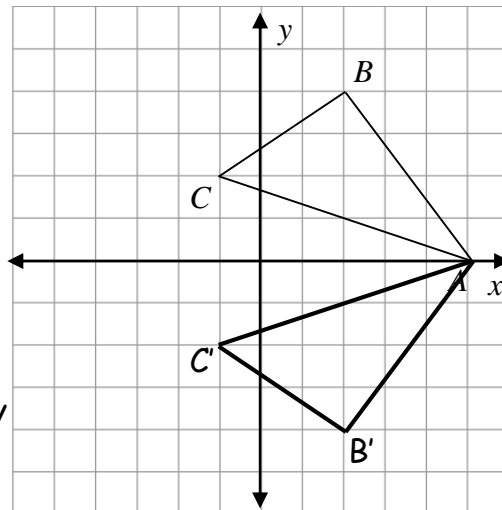reflection in the x-axis.

$A(5, 0) \rightarrow A'(5, 0)$ (didn't move)

$B(2, 4) \rightarrow B'(2, -4)$

$C(-1, 2) \rightarrow C'(-1, -2)$

In general, for a reflection in the x-axis: change the sign of y

$r_{x\text{-}axis}(x, y) = (x, -y)$

b) Draw $\triangle A'B'C'$, the image of $\triangle ABC$ after a
reflection in the y-axis.
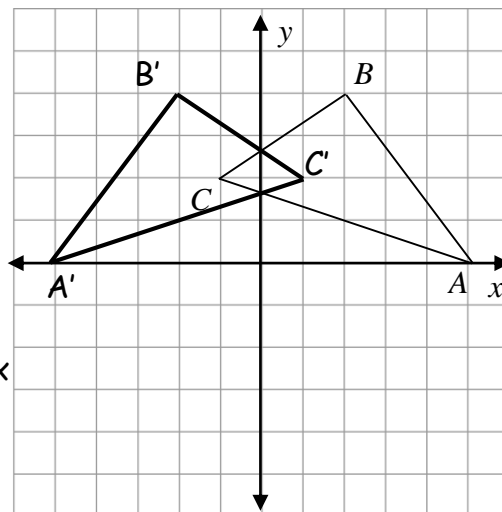
$A(5, 0) \rightarrow A'(-5, 0)$

$B(2, 4) \rightarrow B'(-2, 4)$

$C(-1, 2) \rightarrow C'(1, 2)$

In general, for a reflection in the y-axis: change the sign of x

$r_{y\text{-}axis}(x, y) = (-x, y)$

c) Draw $\triangle A'B'C'$, the image of $\triangle ABC$ after a
reflection in the line $y = x$.

$A(5, 0) \rightarrow A'(0, 5)$

$B(2, 4) \rightarrow B'(4, 2)$

$C(-1, 2) \rightarrow C'(2, -1)$

In general, for a reflection in the line $y = x$: switch x and y

$r_{y=x}(x, y) = (y, x)$